

ALGEBRA Intermediate Revision Sheet

SIMPLIFY (collect like terms).

$$3a + 5b - 7a - b = -4a + 4b$$

EXPAND

Write without brackets

a.

$$3b(5 - b)$$

$$3b \times 5 - 3b \times b$$

$$15b - 3b^2$$



b.

method1 $(2a - 5)(a + 6)$

or method2 $2a(a + 6) - 5(a + 6)$

$$2a \times a \text{ and } +2a \times 6 - 5 \times a \text{ and } -5 \times 6$$

$$2a^2 + 12a - 5a - 30$$

$$2a^2 + 7a - 30$$

FACTORISE

include brackets

(Factorise is the opposite of expand.)

Factorise and simple expression

$$4x^2 + 12x$$

two common factors. **x and 4**

$$4x(x + 3)$$

Simple Quadratic expression

A quadratic is any expression where the highest power of x is x^2 e.g. $x^2 + bx + c$

Quadratics always factorise into the form $(x \pm ?)(x \pm ?)$

Look for two numbers that multiply to make the last number and add to make the middle number

$$x^2 + 6x - 16$$

e.g. multiply to -16 and add to +6 = 8 x -2 and 8 +

(-2)

$$(x + 8)(x - 2)$$

$$x^2 - 11x + 36$$

e.g. multiply to 36 and add to -11 = -9 x -4 and -9 + (-

4)

$$(x - 9)(x - 4)$$

Quadratic Two square numbers

A special quadratic written in the form $a^2 - b^2$. Two square numbers or terms with a 'minus' sign between them. (difference of two squares) Can be factorised in the form $(a - b)(a + b)$

Example 1

$$4x^2 - 9$$

$$a = 2x \quad b = 3$$

$$(2x - 3)(2x + 3)$$

Example 2

$$x^2 - 100$$

$$(x - 10)(x + 10)$$

REARRANGE

To undo an equation you must use opposite operations.

$+$	\leftrightarrow	$-$	$\&$	\div	\leftrightarrow	\times
a^2	\leftrightarrow	\sqrt{a}	$\&$	a^3	\leftrightarrow	$\sqrt[3]{a}$

“when and number or letter Crosses the equal sign Change the sign”

Operators must be applied to **both** sides and **all parts** of the equation.

It is possible to rearrange in any order but care must be taken and usually one way is easier than another

$$2x + 4 = 14 \quad \text{either } \div 2 \text{ or } - 4$$

e.g. If we $\div 2$ first we must divide **all** parts of the equation by 2

$$2x + 4 = 14 \quad (\div 2)$$

$$x + 2 = 7$$

$$x = 7 - 2 \quad (-2)$$

$$x = 5$$

Or - 4 from both sides first

$$2x + 4 = 14 \quad (-4)$$

$$2x = 10 \quad (\div 2)$$

$$x = 5$$

SOLVE

If a question asks you to solve then you must find the value of x.

Rearrange so that x is on one side of the equation and number is on the other

There are three methods used to solve equations, **Expand, Factorise, Trial and Improvement,**

SOLVE by EXPANDING

a. $2(3a + 2) = 7$ Expand

$$6a + 4 = 7$$

$$6a = 3$$

$$a = \frac{1}{2}$$

subtract 4 from both sides
divide by 6

b. $\frac{2x - 3(x + 4)}{5} = -2$ multiply both side by 5

$$2x - 3(x + 4) = -10$$

$$2x - 3x - 12 = -10$$

$$-x - 12 = -10$$

$$-x = 2$$

$$x = -2$$

Expand
Simplify
rearrange

Therefore $x = -2$

TRIAL and IMPROVEMENT

The solution to the equation $x^3 - 7x - 75 = 0$ lies between 4 and 5.

Use the trial and improvement method to find a solution to the equation to 1.d.p.

Therefore the solution is between 4.75 and 4.8 .

The solution to 1.d.p. 4.8

	Value $x^3 - 7x - 75$	
4.5	-15.375	Too small
4.7	-4.077	Too small
4.8	1.992	Too big
4.75	-1.078	Too small

SOLVE THROUGH FACTORISING

Solving a quadratic (A quadratic has two solutions every time)

To solve the equation must be equal to 'zero'. (If $xy = 0$ then $x = 0$ or $y = 0$ or x and $y = 0$)

You must factorise first

$$x^2 + 6x - 16 = 0$$

$$(x + 8)(x - 2) = 0$$

either $(x + 8) = 0$ or $(x - 2) = 0$

Therefore $x = -8$ or $x = 2$

Quadratic Equation The General Formula.

If it is not possible to solve by factorising use the, use the general formula

for any quadratic in the form $ax^2 + bx + c = 0$ the general formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 1.

Use the formula method to solve $3x^2 - 2x - 2 = 0$, giving your answers to 2.d.p. Show your working out
Calculate. from the equation $a = 3$ $b = -2$ $c = -2$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times -2}}{2 \times 3}$$

$$\frac{2 + \sqrt{4 + 24}}{6} \quad \text{or} \quad \frac{2 - \sqrt{4 + 24}}{6}$$

calculate the value inside the brackets first

$$\frac{2 + \sqrt{28}}{6} \quad \text{or} \quad \frac{2 - \sqrt{28}}{6}$$

write **two** equations

$$= 1.22 \quad \text{or} \quad -0.55$$

SIMULTANEOUS EQUATIONS

Steps to Solve. Multiply the equations so that the x values of both equations are equal.

Add or subtract the two new equations from each other Solve and find the value of y

Substitute the 'y' value into one of the original equations to find a value for 'x'

$$2x + 6y = 13 \quad (\text{multiply and } 3)$$

$$3x - 2y = 3 \quad (\text{multiply and } 2)$$

$$6x + 18y = 39 \quad \boxed{1} \quad \text{Subtract one equation from the other}$$

$$6x - 4y = 6 \quad \boxed{2} \quad \text{be careful of the signs}$$

$$\boxed{1} - \boxed{2} \quad 22y = 33 \quad \text{therefore the } = \frac{33}{22} \quad \text{the } = \frac{3}{2}$$

Substitute the 'y' value into either of the original equations to find the value of 'x'

$$2x + 6 \times \frac{3}{2} = 13$$

$$2x + 9 = 13$$

$$2x = 4$$

therefore $x = 2$ ac **the = 1.5**

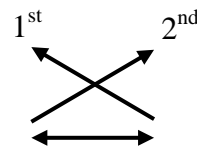
EQUATIONS and EXPRESSIONS that INCLUDE FRACTIONS

Simplify

fractions

$$\frac{5}{4x+2} - \frac{2}{2x-3}$$

Remember the X pattern when adding/subtracting



$$\frac{5(2x-3) - 2(4x+2)}{(4x+2)(2x-3)}$$

Expand the top

leave the bottom in its brackets

$$\frac{10x-15-8x-4}{(4x+2)(2x-3)}$$

Collect terms

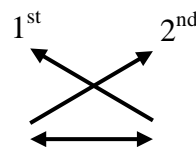
$$\frac{2x-19}{(4x+2)(2x-3)}$$

Solve

fractions

$$\frac{x-2}{3} - \frac{x+3}{2} = 8$$

Remember the X pattern when adding/subtracting



$$\frac{2(x-2) - 3(x+3)}{2 \times 3} = 8$$

Expand the top

$$2x - 4 - 3x - 9 = 48$$

(6 multiplied by 8)

$$-x - 13 = 48$$

Collect terms and solve

$$-x = 61$$

therefore $x = -61$

CHANGE THE SUBJECT

Example 1 In the formula $T = 2\pi\sqrt{\frac{l}{g}}$ T is the subject (written by itself on the left)

Change the formula so that l becomes the subject. Remember opposite operations

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$$

$$\frac{T^2}{4\pi^2} = \frac{l}{g}$$

$$\frac{gT^2}{4\pi^2} = l$$

divide both sides by 2π

Square both sides

multiply both sides by g

Therefore
$$l = \frac{gT^2}{4\pi^2}$$

$x + n =$ change to $-n$

$x - n =$ change to $+n$

$+$	\leftrightarrow	$-$
\div	\leftrightarrow	\times
a^2	\leftrightarrow	\sqrt{a}

$nx =$ divide down

$\frac{x}{n} =$ multiply up

Example 2 Exam question

Make 'e' the subject of the formula $\frac{d(2+e)}{5-e} = 3$

$$d(2+e) = 3(5-e) \quad \text{Multiply both sides by } (5-e)$$

$$2d + de = 15 - 3e \quad \text{Expand brackets}$$

$$de + 3e = 15 - 2d \quad \text{Rearrange so that all the terms with 'e' are all on the left hand side}$$

$$e(d+3) = 15 - 2d \quad \text{Factorise}$$

$$e = \frac{15-2d}{d+3} \quad \text{Divide both sides with } (d+3)$$